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GENETIC AND ENVIRONMENTAL ASPECTS OF THE LINEAR HYPERBOLIC LACTATION CURVE

Evaristo Bianchini-Sobrinho[1] and Francisco A. Moura Duarte[2]

ABSTRACT

The linear hyperbolic lactation curve was used to describe 1084 lactations of Gyr cows. Of the curves traced for each cow, 487 showed that production peaked during the first control and behaved in a practically linear manner. The asymptotes of the other curves gave an estimate of maximum total mean production for each lactation. The parameters of the curve were affected by calving season and number of milkings. The heritabilities of the characteristics of the linear hyperbolic lactation curve were of small magnitude, except for those estimated for production peak, β_1, peak day and coefficient of determination. It is possible to establish a selection criterion for the highest peak value and for the lowest β_1 value in order to increase total milk yield in a lactation.

INTRODUCTION

Many equations are utilized to describe the lactation period of dairy cows, with the gamma curve described by Wood (1967) being the most frequently used. While looking for alternative equations, we showed that the linear hyperbolic curve has a slightly better fit than the gamma curve when used to describe the lactations of Gyr dairy cows (Bianchini Sobrinho, 1984; Bianchini Sobrinho *et al.*, 1986). We also noted that in several cases the shape of the lactation curve was practically linear, with no growth from calving to the production peak. This fact causes parameter b of the

[1] Departamento de Ciências Exatas, Faculdade de Ciências Agrárias e Veterinárias de Jaboticabal, UNESP, 14870 Jaboticabal, SP, Brasil. Send correspondence to E.B.-S.

[2] Departamento de Genética, Faculdade de Medicina de Ribeirão Preto, USP, 14049 Ribeirão Preto, SP, Brasil.

gamma lactation curve to be negative, thus making the curve atypical for the description of a lactation. Using the gamma curve, we detected 36% atypical curves in the herd under study.

Environmental effects can play a predominant role in changing the shape of the lactation curve, but in many studies the authors have evaluated different genetic parameters of the lactation curve. The main objective of the present study was to discuss genetic and non-genetic aspects that may influence the shape of the linear hyperbolic lactation curve for a Gyr herd.

MATERIAL AND METHODS

Data used were 12,879 records of production of 553 Gyr dairy cows considering only the first three lactations in a total of 1084. These cows are daughters of 27 sires from a herd belonging to Sant'Ana da Serra farm, township of Cajuru, State of São Paulo, Brazil, born from 1961 to 1979. The climate of this region is of the subtropical type, with dry winters. Rainfall distribution suggests definition of two calving seasons, a dry one from April to September and a rainy one from October to March.

The linear hyperbolic curve is defined by the equation:

$$Y = \beta_0 + \beta_1 X + \beta_2 1/X,$$

where Y is production at stage X, and β_0, β_1 and β_2 are parameters that determine the shape of the curve (Bianchini, 1984). Parameters of the equation were estimated by a multiple linear regression model of Y on X and 1/X by least squares method. The model may exhibit a peak for values of $X = \sqrt{\beta_2/\beta_1}$ when $\beta_1 < 0$ and $\beta_2 < 0$.

The line $Y = \beta_0 + \beta_1 X$ is the oblique asymptote of the curve and its angular coefficient β_1 can be taken as the rate of decline (in kg) in milk per day, thus supplying a measurement of the persistence of lactation for lactations having the same initial level.

The characteristics studied were β_0, β_1, β_2, peak day, peak value, total yield and coefficient of determination calculated by the following formula:

$$R^2 = \frac{\text{Sum of squares due to regression}}{\text{Total corrected sum of squares}}$$

The general mixed models described by Harvey (1977), used to determine which factors influence the shape of the lactation curve and to calculate the genetic and environmental parameters, are given by the following equations:

$$Y_{ijk} = \mu + a_l + F_K + e_{ijke}$$

$$Y_{ijke} = \mu + a_l + b_{ij} + F_K + e_{ijke}$$

where Y = trait to be studied, a_i = sire effect (random), b_{ij} = within sire cow effect (random), F_k = a set of fixed effects (lactation order, calving season, number of milkings), and e = random error.

Heritability was estimated by the following formula:

$$\hat{h}^2 = \frac{4\,\hat{\sigma}^2_T}{\hat{\sigma}^2_T + \hat{\sigma}^2_e},$$

where $\hat{\sigma}^2_T$ = estimate of sire variance component, and $\hat{\sigma}^2_e$ = estimate of residual variance component.

Environmental correlations were calculated by the following formula:

$$\hat{r}\,(h, h') = \frac{\hat{\sigma}^2_E\,(h, h') + \frac{NW}{NR_1}\,\hat{\sigma}^2_S\,(h, h')}{\hat{\sigma}^2_E\,(h) + \frac{NW}{NR_1}\,\hat{\sigma}^2_S\,(h) + \frac{NW}{NR_1}\,\hat{\sigma}^2_S\,(h')}$$

where $\hat{r}_E$ (h, h') is the environmental correlation between traits h and h', $\hat{\sigma}^2_E$ (h, h') is the estimate of the environmental variance components, $\hat{\sigma}^2_S$ (h, h') is the estimate of sire variance component, NR_1 = 0.25, and NW = 0.75.

Genetic correlations were estimated by dividing the variance components estimated for the set of traits by the geometric mean of variance components estimated for each set.

RESULTS

We obtained 487 linear hyperbolic lactation curves in which maximum production occurred during the first control; all of them showed a practically linear behavior up to the end of lactation (parameter β_2 can be considered statistically null). The remaining 597 curves, constructed starting on the 10th day of lactation, showed an increase from calving to the peak of production and were analyzed for the following characteristics: curve parameters, peak day, peak value, coefficient of determination (R^2) and total production. The values obtained for these characteristics are given in Tables I and II.

Table I shows that the degree of fit increased from the first to the third lactation. Lines Y = 11.29 - 0.01957 X, Y = 15.64 - 0.03232 X and Y = 17.25 - 0.03880 X are the asymptotes of the curve for the first, second and third lactation, respectively. If we sum the values from X = 1 to X = 305 for the curves showing growth from calving to peak production we obtain an estimate of total mean maximum yield through the asymptote, whose values were 2350.21, 3261.98 and 3450.00 kg milk for the first, second and third lactation, respectively. The mean values observed were 2282.96, 3167.04 and 3356.72 kg milk for the first, second and third lactation, respectively.

Table I - Characteristics of the linear hyperbolic curve (means ± SD).

Lactation	Rate of reduction (β_1)	Peak day (days)	Peak value (kg of milk)	R^2
1	-0.01957 ± 0.011	46.76 ± 22.29	9.48 ± 2.52	0.61 ± 0.27
2	-0.03232 ± 0.013	35.30 ± 18.52	13.33 ± 3.02	0.79 ± 0.17
3	-0.03880 ± 0.014	30.38 ± 14.56	14.89 ± 3.37	0.83 ± 0.18

Table II - Parameters estimated for the linear hyperbolic curve (means ± SD) starting from the 10th day of lactation for each lactation.

Lactation	β_0	β_1	β_2	N
1	11.29 ± 3.32	-0.01957 ± 0.011	-50.28 ± 56.02	290
2	15.64 ± 3.92	-0.03232 ± 0.013	-51.28 ± 59.31	179
3	17.25 ± 3.94	-0.02880 ± 0.014	-43.87 ± 45.89	128

Heritabilities (mean ± SEM) estimated for the parameters and the characteristics of the linear hyperbolic curve are given in Table III. Heritabilities for β_1, peak day, and peak value, even though they had relatively high standard errors, were high in relation to the remaining characteristics. Since β_1 is the rate of decrease in milk (kg

Table III - Heritability estimates (means ± SEM) for the parameters and characteristics of the linear hyperbolic curve per lactation.

Parameters	Lactation 1	Lactation 2	Lactation 3
	h^2	h^2	h^2
β_0	0.05 ± 0.10	0.32 ± 0.21	0.10 ± 0.10
β_1	0.22 ± 0.23	0.30 ± 0.21	0.12 ± 0.10
β_2	0.16 ± 0.12	– [a]	0.10 ± 0.10
Peak day	0.66 ± 0.36	0.35 ± 0.21	0.10 ± 0.10
Peak value	0.27 ± 0.25	0.47 ± 0.22	– [a]
R^2	0.30 ± 0.28	0.90 ± 0.26	0.40 ± 0.27

[a] Some heritabilities were not estimated because the sire variance component obtained was negative.

per day) after the peak of production, the h^2 values obtained for the first and second lactations indicate that this parameter has the potential to be used as a selection criterion.

The genetic and phenotypic correlations of the traits and parameters obtained through the linear hyperbolic curve per lactation are given in Tables IV through VI. Phenotypically, β_0 was positively correlated with peak day, peak value, R^2 and total yield, and negatively correlated with β_1 and β_2. Genetically, the correlations involving β_2 were not consistent over the three lactations, though β_2 was positively correlated with peak value and total yield in the second lactation. Phenotypically, β_1 was positively correlated with β_2 and total yield and negatively correlated with β_0 in all three

Table IV - Genetic correlation above the diagonal and phenotypic correlation below the diagonal of the linear hyperbolic curve for first lactation.

Parameters	I	II	III	IV	V	VI	VII
I β_0	–	0.25	0.74	-0.94	a	0.33	a
II β_1	-0.81	–	a	a	-0.27	a	0.45
III β_2	-0.50	0.48	–	-0.77	0.51	0.39	0.51
IV Peak day	0.08	-0.04	-0.71	–	-0.61	a	0.29
V Peak value	0.87	-0.58	-0.05	-0.24	–	0.10	0.77
VI R^2	0.35	-0.48	-0.11	-0.20	-0.20	–	-0.85
VII Total P	0.45	0.09	0.08	-0.04	0.29	0.02	–

[a] Genetic correlations could not be estimated because sire variance components were negative.

Table V - Genetic correlation above the diagonal and phenotypic correlation below the diagonal of the linear hyperbolic curve for second lactation.

Parameters	I	II	III	IV	V	VI	VII
I β_0	–	-0.72	a	0.17	0.95	0.23	0.34
II β_1	-0.66	–	a	0.45	-0.37	-0.69	0.49
III β_2	-0.57	0.37	–	a	a	a	a
IV Peak day	0.30	0.01	-0.80	–	0.04	-0.78	0.38
V Peak value	0.86	-0.45	-0.13	-0.06	–	0.01	0.75
VI R^2	0.09	-0.36	0.06	-0.31	0.08	–	-0.55
VII Total P	0.50	0.24	-0.12	0.22	0.66	-0.15	–

[a] Genetic correlations could not be estimated because sire variance components were negative.

Table VI - Genetic correlation above the diagonal and phenotypic correlation below the diagonal of the linear hyperbolic curve for third lactation.

Parameters	I	II	III	IV	V	VI	VII
I β_0	–	a	a	a	a	a	a
II β_1	-0.59	–	a	a	a	a	a
III β_2	-0.40	0.35	–	a	a	a	a
IV Peak day	0.23	-0.02	-0.85	–	a	a	a
V Peak value	0.84	-0.39	-0.02	-0.10	–	a	a
VI R^2	0.31	-0.46	-0.09	-0.05	0.26	–	a
VII Total P	0.61	0.17	-0.02	0.23	0.11	0.10	–

[a]Genetic correlations could not be estimated because sire variance components were negative.

lactations. The correlations with the remaining traits were not consistent. Genetically, β_1 was correlated with total yield in the first two lactations, whereas the correlations with the remaining traits were not consistent. β_1 may have potential for selection in the first and second lactations, with a consequent increase in total yield.

Peak value was genetically and phenotypically correlated in a positive manner with total yield in the first two lactations and negatively correlated with β_1. Selection for a greater peak value would reduce the rate of decrease, with a consequent increase in total yield. The correlations involving the remaining traits both in genetic and phenotypic terms were inconsistent over lactations.

Table VII shows analysis of variance for the parameters of the linear hyperbolic curve for the first lactation. It may be seen that parameters β_1 and β_1 were

Table VII - Analysis of variance of the parameters of the linear hyperbolic lactation curve for first lactation.

Sources of variation	d.f.	Parameter mean squares		
		β_0	β_1	β_2
Sire	25	7.50	1.45×10^{-4}	3554.04
Calving season	1	7.16	1.79×10^{-4}*	18694.94*
Number of milkings	1	1000.04***	52.86×10^{-4}**	25456.09*
Residue	262	6.92	1.12×10^{-4}	2894.29

*$P < 0.05$; **$P < 0.01$; ***$P < 0.001$.

affected by calving season and number of milkings, and parameter β_0 was highly affected by number of milkings. This means that the shape of the linear hyperbolic lactation curve is affected by calving season and number of milkings during first lactation. The analyses for the remaining lactations showed similar results. During the first lactation, cows that had calved during the dry season produced 2954.34 kg milk versus 2854.37 kg milk produced by cows that had calved during the rainy season. Higher yields by cows that had calved during the dry season were also observed in the remaining lactations.

CONCLUSIONS

Of the 1084 lactations analyzed, 487 (44.9%) showed production peaks during the first record. For these lactations the behavior was practically linear and parameter β_2 of the curve can be discarded.

The adymptotes of the curves showing growth from first record to lactation peak supply an estimate for mean total maximum production during one lactation.

The parameters of the curve are affected by calving season and number of milkings.

Heritabilities of the linear hyperbolic lactation curve traits were of small magnitude and had a very high standard error, except for those estimated for production peak, β_1, peak day, and R^2.

It is possible to establish a selection criterion for highest peak value of the linear hyperbolic lactation curve in order to increase total yield.

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RESUMO

A curva de lactação linear hiperbólica foi utilizada para descrever 1084 lactações de vacas da raça Gir. Mostrou-se que dentre as curvas traçadas para cada animal, 487 delas possuiam pico de produção no primeiro controle, e tinham um comportamento praticamente linear. As assíntotas das demais curvas proporcionam uma estimativa da produção total média máxima para cada lactação. Os parâmetros da curva foram influenciados pela estação de parição e número de ordenhas. As herdabilidades das características da curva de lactação linear hiperbólica são de pequena grandeza com exceção das estimadas para o pico de produção, β_1, dia do pico e coeficiente de determinação. É possível estabelecer um critério de seleção para o maior valor do pico e para menor β_1, visando o aumento da produção total de leite em uma lactação.

REFERENCES

Bianchini-Sobrinho, E. (1984). Estudo da curva de lactação de vacas da raça Gir. Doctoral Thesis. Faculdade de Medicina de Ribeirão Preto, USP.

Harvey, W.R. (1977). User's Guide for LSML-76: Mixed Model Least Squares and Maximum Likelihood Computer Program. Ohio State University, 76 pp.

Wood, P.D.P. (1967). Algebraic models of the lactation curve in cattle. *Nature 216*: 164-165.

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